



**Mathematics  
Higher level  
Paper 2**

Wednesday 11 May 2016 (morning)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

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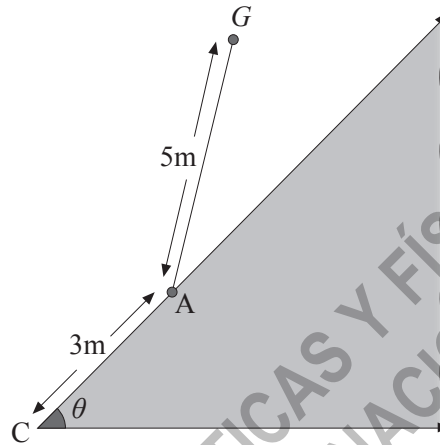




4. [Maximum mark: 6]

The diagram below shows a fenced triangular enclosure in the middle of a large grassy field. The points A and C are 3 m apart. A goat G is tied by a 5 m length of rope at point A on the outside edge of the enclosure.

Given that the corner of the enclosure at C forms an angle of  $\theta$  radians and the area of field that can be reached by the goat is  $44 \text{ m}^2$ , find the value of  $\theta$ .



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Turn over

















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### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 22]

Let  $f(x) = x^4 + 0.2x^3 - 5.8x^2 - x + 4$ ,  $x \in \mathbb{R}$ .

(a) Find the solutions of  $f(x) > 0$ . [3]

(b) For the curve  $y = f(x)$ .

(i) Find the coordinates of both local minimum points.

(ii) Find the  $x$ -coordinates of the points of inflexion. [5]

The domain of  $f$  is now restricted to  $[0, a]$ .

(c) (i) Write down the largest value of  $a$  for which  $f$  has an inverse. Give your answer correct to 3 significant figures.

(ii) For this value of  $a$  sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  on the same set of axes, showing clearly the coordinates of the end points of each curve.

(iii) Solve  $f^{-1}(x) = 1$ . [6]

Let  $g(x) = 2 \sin(x-1) - 3$ ,  $-\frac{\pi}{2} + 1 \leq x \leq \frac{\pi}{2} + 1$ .

(d) (i) Find an expression for  $g^{-1}(x)$ , stating the domain.

(ii) Solve  $(f^{-1} \circ g)(x) < 1$ . [8]



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12. [Maximum mark: 16]

Consider the curve,  $C$  defined by the equation  $y^2 - 2xy = 5 - e^x$ . The point  $A$  lies on  $C$  and has coordinates  $(0, a)$ ,  $a > 0$ .

- (a) Find the value of  $a$ . [2]
- (b) Show that  $\frac{dy}{dx} = \frac{2y - e^x}{2(y - x)}$ . [4]
- (c) Find the equation of the normal to  $C$  at the point  $A$ . [3]
- (d) Find the coordinates of the second point at which the normal found in part (c) intersects  $C$ . [4]
- (e) Given that  $v = y^3$ ,  $y > 0$ , find  $\frac{dv}{dx}$  at  $x = 0$ . [3]

13. [Maximum mark: 22]

Six balls numbered 1, 2, 2, 3, 3, 3 are placed in a bag. Balls are taken one at a time from the bag at random and the number noted. Throughout the question a ball is always replaced before the next ball is taken.

- (a) A single ball is taken from the bag. Let  $X$  denote the value shown on the ball. Find  $E(X)$ . [2]
- (b) Three balls are taken from the bag. Find the probability that
- (i) the total of the three numbers is 5;
- (ii) the median of the three numbers is 1. [6]
- (c) Ten balls are taken from the bag. Find the probability that less than four of the balls are numbered 2. [3]
- (d) Find the least number of balls that must be taken from the bag for the probability of taking out at least one ball numbered 2 to be greater than 0.95. [3]
- (e) Another bag also contains balls numbered 1, 2 or 3. Eight balls are to be taken from this bag at random. It is calculated that the expected number of balls numbered 1 is 4.8, and the variance of the number of balls numbered 2 is 1.5.
- Find the least possible number of balls numbered 3 in this bag. [8]



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