Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 11 May 2016 (morning)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

13 pages

2216 – 7206

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

ABCD is a quadrilateral where $AB = 6.5$, $BC = 9.1$, $CD = 10.4$, $DA = 7.8$ and $C\hat{D}A = 90^\circ$. Find $A\hat{B}C$, giving your answer correct to the nearest degree.

2. [Maximum mark: 7]

A random variable X is normally distributed with mean 3 and variance 2^2.

(a) Find $\mathrm{P}(0 \le X \le 2)$. [2]

(b) Find $\mathrm{P}(|X| > 1)$. [3]

(c) If $\mathrm{P}(X > c) = 0.44$, find the value of c. [2]

3. [Maximum mark: 6]

Solve the simultaneous equations

$$\ln \frac{y}{x} = 2$$

$$\ln x^2 + \ln y^3 = 7\,.$$

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4. [Maximum mark: 6]

The sum of the second and third terms of a geometric sequence is 96.

The sum to infinity of this sequence is 500.

Find the possible values for the common ratio, r.

5. [Maximum mark: 6]

The function f is defined as $f(x) = \sqrt{\dfrac{1-x}{1+x}}$, $-1 < x \le 1$.

Find the inverse function, f^{-1} stating its domain and range.

6. [Maximum mark: 8]

A company produces rectangular sheets of glass of area 5 square metres. During manufacturing these glass sheets flaws occur at the rate of 0.5 per 5 square metres. It is assumed that the number of flaws per glass sheet follows a Poisson distribution.

(a) Find the probability that a randomly chosen glass sheet contains at least one flaw. [3]

Glass sheets with no flaws earn a profit of $\$5$. Glass sheets with at least one flaw incur a loss of $\$3$.

(b) Find the expected profit, P dollars, per glass sheet. [3]

This company also produces larger glass sheets of area 20 square metres. The rate of occurrence of flaws remains at 0.5 per 5 square metres.
A larger glass sheet is chosen at random.

(c) Find the probability that it contains no flaws. [2]

7. [Maximum mark: 8]

Consider the curve with equation $x^3 + y^3 = 4xy$.

(a) Use implicit differentiation to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4y - 3x^2}{3y^2 - 4x}$. [3]

The tangent to this curve is parallel to the x-axis at the point where $x = k$, $k > 0$.

(b) Find the value of k. [5]

8. [Maximum mark: 6]

A particle moves such that its velocity $v\,\text{ms}^{-1}$ is related to its displacement s m, by the equation $v(s) = \arctan(\sin s)$, $0 \le s \le 1$. The particle's acceleration is $a\,\text{ms}^{-2}$.

(a) Find the particle's acceleration in terms of s. [4]

(b) Using an appropriate sketch graph, find the particle's displacement when its acceleration is $0.25\,\text{ms}^{-2}$. [2]

9. [Maximum mark: 8]

OACB is a parallelogram with $\vec{OA} = \boldsymbol{a}$ and $\vec{OB} = \boldsymbol{b}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are non-zero vectors.

(a) Show that

(i) $|\vec{OC}|^2 = |\boldsymbol{a}|^2 + 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$;

(ii) $|\vec{AB}|^2 = |\boldsymbol{a}|^2 - 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$. [4]

(b) Given that $|\vec{OC}| = |\vec{AB}|$, prove that OACB is a rectangle. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

A continuous random variable T has probability density function f defined by

$$f(t) = \begin{cases} \dfrac{t|\sin 2t|}{\pi}, & 0 \le t \le \pi \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $y = f(t)$. [2]

(b) Use your sketch to find the mode of T. [1]

(c) Find the mean of T. [2]

(d) Find the variance of T. [3]

(e) Find the probability that T lies between the mean and the mode. [2]

(f) (i) Find $\int_0^T f(t)\,\mathrm{d}t$ where $0 \le T \le \dfrac{\pi}{2}$.

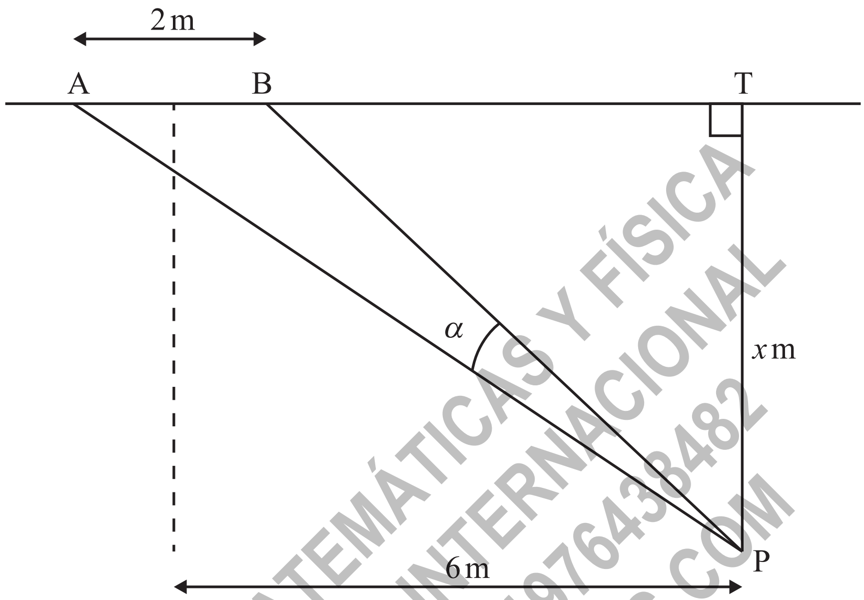
(ii) Hence verify that the lower quartile of T is $\dfrac{\pi}{2}$. [5]

Turn over

Do **not** write solutions on this page.

11. [Maximum mark: 22]

Points A, B and T lie on a line on an indoor soccer field. The goal, $[\mathrm{AB}]$, is 2 metres wide. A player situated at point P kicks a ball at the goal. $[\mathrm{PT}]$ is perpendicular to (AB) and is 6 metres from a parallel line through the centre of $[\mathrm{AB}]$. Let PT be x metres and let $\alpha = \mathrm{A\hat{P}B}$ measured in degrees. Assume that the ball travels along the floor.



(a) Find the value of α when $x = 10$. [4]

(b) Show that $\tan\alpha = \dfrac{2x}{x^2 + 35}$. [4]

The maximum for $\tan\alpha$ gives the maximum for α.

(c) (i) Find $\dfrac{\mathrm{d}}{\mathrm{d}x}(\tan\alpha)$.

(ii) Hence or otherwise find the value of α such that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\tan\alpha) = 0$.

(iii) Find $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}(\tan\alpha)$ and hence show that the value of α never exceeds 10°. [11]

(d) Find the set of values of x for which $\alpha \geq 7^\circ$. [3]

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12. [Maximum mark: 23]

The functions f and g are defined by

$$f(x) = \frac{e^x + e^{-x}}{2}, x \in \mathbb{R}$$

$$g(x) = \frac{e^x - e^{-x}}{2}, x \in \mathbb{R}$$

(a) (i) Show that $\frac{1}{4f(x) - 2g(x)} = \frac{e^x}{e^{2x} + 3}$.

(ii) Use the substitution $u = e^x$ to find $\int_0^{\ln 3} \frac{1}{4f(x) - 2g(x)} dx$. Give your answer in the form $\frac{\pi\sqrt{a}}{b}$ where $a, b \in \mathbb{Z}^+$. [9]

Let $h(x) = nf(x) + g(x)$ where $n \in \mathbb{R}, n > 1$.

(b) (i) By forming a quadratic equation in e^x, solve the equation $h(x) = k$, where $k \in \mathbb{R}^+$.

(ii) Hence or otherwise show that the equation $h(x) = k$ has two real solutions provided that $k > \sqrt{n^2 - 1}$ and $k \in \mathbb{R}^+$. [8]

Let $t(x) = \frac{g(x)}{f(x)}$.

(c) (i) Show that $t'(x) = \frac{[f(x)]^2 - [g(x)]^2}{[f(x)]^2}$ for $x \in \mathbb{R}$.

(ii) Hence show that $t'(x) > 0$ for $x \in \mathbb{R}$. [6]

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